Examining Headway Distribution Models Using Urban Freeway Loop Event Data

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ABSTRACT

Vehicle headway distribution is fundamental for several important traffic research and simulation issues. Many headway models have been developed over the past decades. Each of them has its own strength and weakness. Selection of the most suitable model for a certain traffic condition remains an open issue. This paper presents a comprehensive study on performance of typical headway distribution models on urban freeways. Using the Advanced Loop Event Data Analyzer (ALEDA) system, a large amount of accurate headway observations were obtained from Interstate Highway 5 in the Seattle area. These headway data were used to calibrate and examine the performance of various headway models. The goodness of fit for several most commonly used headway distribution models were investigated using headways observed on regular lanes and HOV lanes from different time periods of day. In order to evaluate the performance of these headway models, the analytical Kolmogorov-Smirnov test statistic and visualized comparison curves were used to measure and reflect their overall goodness-of-fit to the collected headway data. Although each model has its own practicability to a certain extent, the test results showed that the Double Displaced Negative Exponential Distribution model provided the best fit to our urban freeway headway data, especially, for HOV lanes at wide-ranging flow levels. The shifted lognormal distribution also fits the general-purpose-lane headways very well. As a by product, a new standard parameter estimation method was developed for calibrating complex multi-parameter headway models.

Key words*:* *time headway distribution, loop event data, single and mixed models, Kolmogorov-Smirnov goodness-of-fit test, and Q-Q plot.*

1. INTRODUCTION

Vehicle headway is a measure of the temporal space between two vehicles, and is defined as: the elapsed time between the arrival of the leading vehicle and the following vehicle at a designated test point. It is usually measured in seconds. Since the average of vehicle headways is the reciprocal of flow rate, vehicle headways represent microscopic measures of flows passing a point. To some extent, the minimum acceptable mean headway determines the roadway capacity. Accurate modeling and analysis of vehicle headway distribution helps traffic engineers to maximize roadway capacity and minimize vehicle delays. Additionally, vehicle headway distribution is closely related to vehicle merging and permitted left-turn movements. It is essential for determining the capacity of roundabouts and intersections as well as adjusting or coordinating signal timing plans at the signalized intersections. Furthermore, proper headway models can be applied to generate vehicle events in microscopic simulation models and to traffic safety analysis. All these indicate that vehicle headway distribution is fundamental for many traffic flow research and simulation issues.

Many headway models have been developed over the past decades. In general, these models can be classified into two categories: single statistical distribution models and mixed models of two or more distributions. The mixed models are more flexible to represent headways by decomposing them into following and free-following components, but the calibration process may be too complex for field application. In practice, selection of the most suitable headway distribution for a certain traffic condition remains an open issue. Also, most previous headway models focused only on general-purpose-lane traffic. Since High Occupancy Vehicle (HOV) lane traffic typically associates with different traffic compositions and flow characteristics, the applicability of the existing headway models remains unclear. This study, therefore, focuses on examining several commonly used headway distribution models using the headway data observed from both a general purpose lane and an HOV lane of Interstate Highway 5 (I-5) in the Seattle area. During this process, a new method for model calibration was also developed to alleviate the workload for parameter estimation in mixed distribution models of vehicle headways.

This paper is organized as follows. In Section Two, typical headway models and their service conditions are briefly described. This is followed by an introduction to the Advanced Loop Event Data Analyzer (ALEDA) system in Section Three. All headway data used in this study were collected by ALEDA. In Section Four, we introduce a new method developed in this study for parameter estimation. Then, typical headway models were examined in Section Five, and profound discussion on the performance of each model is described. The final section concludes this research effort and proposes further research work.

2. MODEL DESCRIPTION AND ANALYSIS

2.1 Single Distribution Models

Several single distribution models of vehicle headway have been investigated by a number of researchers. Representatives of such single distribution models are exponential distribution *(1)*, normal distribution, Gamma distribution and lognormal distribution models *(2)*. For instance, lognormal distribution is proposed to model headways under car-following situations. A major assumption for lognormal headway models is that a vehicle maintains a safe distance while following its leading vehicle closely at variable speeds. So a headway between two consecutive vehicles may change a random portion of its original value in a short time interval *(2, 3)*. The change of headway can be expressed as a fraction of the current headway:

$$V_i - V_{i-1} = V_{i-1} * \lambda_i \tag{1}$$

where V_i is the headway at the *ith* interval, λ_i is the ratio of headway change (V_i-V_{i-1}) to V_{i-1}. Suppose λ_i is small enough at each interval, then computing the cumulative sum of λ_i yields:

$$\sum_{i=1}^{n} \lambda_i = \sum_{i=1}^{n} \frac{V_i - V_{i-1}}{V_{i-1}} \sim \int_{V_1}^{V_n} \frac{dV}{V} = \ln V_n - \ln V_1 \tag{2}$$

Rearranging terms of Equation (2) results in

$$\ln V_n = \ln V_1 + \sum_{i=1}^{n} \lambda_i \tag{3}$$

Based on the Central Limit Theorem, $\sum_{i=1}^{n} \lambda_i$ is normally distributed, and then V_n is Lognormally distributed *(4)*. However, no appropriate theory supports the assumption that vehicle's arrival patterns obey the law of proportionate effect, e.g. the change proportion of headway λ_i is randomly related to the headway at the *(i-1)th* interval. This assumption can be easily violated in complex traffic conditions.

Gamma distribution is another single headway model widely used due to its flexibility and compatibility. As a family of curves, Gamma distribution may evolve into its two child-distributions: Chi-square and Exponential distribution when one of the two parameters is fixed with specific values.

Although single distribution models are simple and easy to apply, their performance is not satisfactory due to their limited capabilities in approximating small headways. To improve the accuracy of single distribution models, shifted single distribution models are introduced. An example of such models is the Cowan M2 model *(1)* which is a shifted exponential distribution. In this study, the shifted lognormal distribution and Gamma distribution models with a minimum headway of 0.35 sec are further investigated and the parameters are calibrated by Maximum Likelihood Estimation (MLE). Details of these investigations can be found in Sections Four and Five.

2.2 Mixed Distribution Models

Some mixed distribution models are proposed on the assumption that a headway H consists of two components, $H = T + U$, where T is the "tracking or following" component and U is the "free" component. When T is equal to a constant τ and U follows exponential distribution, the mixed model becomes the Cowan M2 *(1)*; when $\tau = 0$, it is reduced to single exponential distribution. Conversely, generalizing these two components result in a series of important models including Cowan M3, M4, the Generalized Queuing Model, and the Semi-Poisson Model *(5)*. Different from those models, Griffiths and Hunt *(6)* proposed another mixed model called Double Displaced Negative Exponential Distribution (DDNED). In this paper, we investigate and compare these typical models on the basis of theoretical analyses and field tests.

Cowan M3

Cowan M3 has been investigated and applied widely *(7, 8, 9)*. It assumes that a proportion θ of vehicles are tracking their predecessors at a constant headway τ and the remaining portion of vehicles $(1-\theta)$ are traveling freely at some headway greater than τ. The free-following component U is assumed to be exponentially distributed *(10, 11)*. Its cumulative distribution function is expressed in Equation (4).

$$F(t)=\begin{cases}1-(1-\theta)\exp(-\gamma(t-\tau), & t\geq\tau\\ 0 & t<\tau\end{cases} \quad (4)$$

where τ is the minimum headway between successive vehicles, θ is the proportion of vehicles tracking at the minimum headway τ, and γ is the exponential decay constant. This model has played an important role because of its simplicity and approximation in describing larger headways. If the proportion of tracking vehicles, θ is equal to zero, the Cowan M3 reduces to the Cowan M2 and if the tracking headway is assumed a general distribution, it is generalized further to Cowan M4.

Cowan M4 and Generalized Queuing Model

Because Cowan M3 cannot model short headways accurately and adequately, a more general model Cowan M4 was introduced. Its cumulative density function is expressed as follows

$$F(t)=\begin{cases}\theta * G(t)+(1-\theta)\int_0^t G(t-u)*\gamma*\exp(-\gamma*u)du & t\geq 0\\ 0 & t<0\end{cases} \quad (5)$$

where $G(t)$ is the general distribution instead of the constant headway τ in Cowan M3. Various known probability models have been tested against field data as the general distribution $G(t)$, such as Gamma distribution, normal distribution and lognormal distribution *(12)*.

Similarly, Branston *(13)* introduced another model known as Generalized Queuing Model (GQM). This model is analogous to the traffic queuing model and its probability density function is given by Branston as follows *(13)*,

$$f(t)=\theta * g(t)+(1-\theta)*\gamma*\exp(-\gamma*t)*\int_0^t g(x)*\exp(\gamma*x)dx \quad (6)$$

Actually, GQM is consistent with Cowan M4 in mathematical nature. By calculating the differential of the cumulative density function in Equation (5), the probability density function for Cowan M4 is obtained as,

$$f(t)=dF(t)/dt=\theta * g(t)+(1-\theta)*\partial(\int_0^t G(t-u)*\gamma*\exp(-\gamma*u)du)/\partial t \quad (7)$$

where, $g(t)=\frac{dG(t)}{dt}$, and to calculate the partial differential of the integral component with variable substitution: $u=t-x$ in Equation (7),

$$\begin{aligned}&\partial(\int_0^t G(t-u)*\gamma*\exp(-\gamma*u)du)/\partial t\\ &=\int_0^t g(t-u)*\gamma*\exp(-\gamma*u)du+G(0)*\exp(-t*\gamma)*\gamma\\ &=\int_0^t g(x)*\gamma*\exp(-\gamma*(t-x))dx\end{aligned} \quad (8)$$

Because the minimum value of headways must be greater than 0, the cumulative density function of following headways $G(0)=0$. By comparing Equation (6), (7) and (8), we can draw a conclusion that GQM is the same as Cowan M4 mathematically. As Branston noted *(13)*, the traffic arrival pattern could be assumed to be a typical queuing system with a single sever (M/G/1) with a random input. The service time is analogous to the following vehicle headways T, the probability θ is compared with the situation that the server is busy. The following vehicle headways are supposed to be independent of the free-following vehicle headways. In this paper, Cowan M4 is further investigated. Several underlying distributions, such as Normal and Gamma distribution are exploited as following headways.

Double Displaced Negative Exponential Distribution (DDNED)

The mixed headway model of Double Displaced Negative Exponential Distribution (DDNED) is proposed by Griffiths and Hunt *(6)*. Compared to the simple displaced negative exponential distribution, the model was found to yield a good fit to their collected data. The probability density function is demonstrated as follows:

$$f(t) = \begin{cases} \phi * \gamma_1 * \exp(-\gamma_1 * (t-d)) + (1-\varphi) * \gamma_2 * \exp(-\gamma_2 * (t-d)) & t \geq d \\ 0 & t < d \end{cases} \quad (9)$$

where $f(t)$ is the probability density function; ϕ is a weighting factor constrained by $0 < \phi \leq 1$; γ_1 and γ_2 are constants associated with the flow status; and d is a displaced parameter.

It has been a challenge to calibrate the four parameters using field observed headways. In previous studies the four parameters in Equation (9) were estimated by some hybrid methods. In this paper, we propose a new method that estimates these parameters.

Other Headway Models

A semi-Poisson Model was proposed by Buckley *(5)* to describe the headway distribution. Wasielewski *(14)* analyzed various characteristics of the semi-Poisson model. As Branston *(13)* stated, the main difference between the semi-Poisson model and Cowan M4 (GQM) is the discrepancy in fitting headways under free flow conditions. Because the explicit expression for the Laplace transform of the following-vehicle headway distribution is required, the use of the semi-Poisson model has been rather limited.

In addition to the above classical headway models, researchers also studied headway distribution from physical perspective. For example, Milan Krbalek *(15)* developed a headway distribution model using the Random Matrix Theory (RMT). However, due to these models' more complicated nature, they are not extensively applied.

3. DATA COLLOCATION

Superior to the previous research work, a more accurate event data collection system, e.g. the Advanced Loop Event Data Analyzer (ALEDA) system *(16)*, was applied for collecting accurate headway data. The ALEDA system was developed at the Smart Transportation Applications and Research Laboratory (STAR Lab) in the University of Washington. It is a portable event data collection and real-time analysis system installed on a laptop computer. The system is capable of polling and storing high-resolution traffic data to preserve individual vehicle information from the inductive loop detectors deployed widely in the existing roadway infrastructure. In our study the event data was collected and recorded at 60 Hz, and individual vehicle information such as presence time, arrival time, and departure time can be extracted from loop event data. By applying the ALEDA system, accuracy of collected headway data can reach 1/30 second, e.g. less than 0.03 second. The accurate headway data enable further investigation and robust estimation of the composite headway models, and adequately reflect characteristics of headway data at various flow levels, especially, the short headways.

Assisted by ALEDA, vehicle headway data were collected at the ES-167D station on I-5. There are four southbound lanes at this station. In order to minimize estimation errors, the collected headway data were cleaned for their reliability in advance. A total of 24 hours of regular weekday headway data were continuously collected from the loop detector station. These data are used to examine the performance of several commonly used headway models during different time periods of day, including a morning period (7:00 to 9:00 am), a noon period (11:00am-13:00pm), an evening period (4:00-7:00pm), and a low-volume night period (2:00 to 5:00 am). Specifically, the headway data from one regular lane (Lane 2) and one particular HOV lane (Lane 4) were used. Selection of these two lanes is because Lane 2 represents typical traffic patterns on general purpose lanes and Lane 4 reflects a different traffic pattern observable on HOV lanes. Using data from both Lane 2 and Lane 4 in our tests further enables us to examine the robustness and reliability of each headway model under various traffic conditions. Traffic flow rates of different periods varied widely from 24 to 1680 vehicles per hour (vph). The fundamental statistical characteristics of the headway data are shown in Table 1.

TABLE 1 Fundamental Statistical Analysis of the Collected Headways on Multi-lane Freeways

	Lane 2 (Regular)				Lane 4 (HOV)			
Period	Morning	Noon	Evening	Night	Morning	Noon	Evening	Night
Average Flow Rate (vph)	1680	1485	1445	323	1408	430	462	24
Sample Size	3360	2971	4335	969	2817	859	1385	71
Mean of Headways (Sec)	2.143	2.423	2.490	11.145	2.556	8.385	7.795	153.773
Std deviation of Headways (Sec)	1.167	1.862	1.897	13.970	2.3656	9.457	9.616	349.714
Minimum Value (Sec)	0.453	0.354	0.375	0.421	0.453	0.453	0.422	0.750
Maximum Value (Sec)	11.125	18.719	18.890	130.015	25.750	49.219	97.328	1848.703

4. PARAMETER ESTIMATION AND CALIBRATION

Parameters of headway distribution models must be properly estimated before these models can be applied. The goodness of fit for them is significantly affected by the quality of the estimated parameters. For single distribution models, standard parameter estimation technologies can be applied to achieve good estimates for the parameters. In our study, the Maximum Likelihood Estimation (MLE) technique is used to formulate the best unbiased estimators.

For mixed distribution models, parameter estimation becomes more difficult due to the complicated structures of the probability density functions. The mixed distribution models examined in this study are DDNED, Cowan M4 with Gamma distributed following-vehicle headway, and Cowan M4 with normal distributed following vehicle headway. Each of these models has four unknown parameters to estimate. Several conventional methods were used to estimate the parameters, including Moment Estimation (ME), MLE, and the hybrid method that integrates the moment estimate with the minimized Chi-squared function *(1, 8, 14)*. However, these techniques may not be appropriate due to the complicated interactive associations among the four parameters. For instance, the four nonlinear differential equations derived from MLE could not be easily solved even numerically to obtain the corresponding parameters. The method of ME produces an easier equation group to handle numerically, but the estimators are not reliable because of the large variances of the third and fourth sample moments. The alternative hybrid method combines ME with the minimized Chi-squared statistic for parameter estimation *(6)*. The simplified procedure of the hybrid method, however, is still too complicated to apply. Hoogendoorn *(12)* developed an estimation method for the GQM on the basis of the minimization of a mean integrated squared error distance in the frequency domain. Because this method requires an explicit expression for the Fourier Transformation of the following-vehicle headway distribution, its applicability is limited to some extent.

In this study, we propose a simple yet effective estimation method for calibrating mixed distribution models with four or more parameters. The main idea is to construct an objective function to minimize the Sum of the Squared Errors (SSE) between estimated and empirical distribution functions; then to calculate the optimal solutions and obtain parameter estimates. The objective function is expressed as follows:

$$\min SSE = \sum_{i=1}^{n} (f(x_i) - y_i)^2 \tag{10}$$

Where, i is the time series index, x_i is the measured headway data, y_i is the observed probability density function, $f(x_i)$ is the corresponding estimated probability density function. The nature of the optimization objective is to calculate the most suitable parameters for the assumed headway distribution model and to produce the best fits to the data by minimizing SSEs.

Now we use the Cowan M4 model with Gamma distributed following-vehicle headway as an example to illustrate the process. Based on Equation (5), the estimated probability density function can be expressed as:

$$\begin{aligned} f(x) &= \theta * \frac{x^{\alpha-1} e^{-x/\beta}}{\Gamma(\alpha)\beta^{\alpha}} + \\ &(1-\theta) * \frac{-\beta^{-\alpha} x^{\alpha} e^{-\lambda x} \lambda(-1+1/\beta\lambda)^{1-\alpha} \left(\Gamma(\alpha) - \dfrac{\int_0^{-x\lambda(1-1/\beta\lambda)} t^{-x\lambda(1-1/\beta\lambda)-1} e^{-t} dt}{\Gamma(-x\lambda(1-1/\beta\lambda))}\right)}{(1-1/\beta\lambda)\Gamma(\alpha)} \end{aligned} \tag{11}$$

where, α and β are the shape and scale parameters of Gamma distribution, respectively; $\Gamma(\alpha)$ is the Gamma function. Many algorithms have been developed for nonlinear optimization problems. Of these algorithms, the Trust-Region method is recognized as a simple yet powerful one *(17, 18)*. The basic idea of the Trust-Region algorithm is to approximate the objective function at a certain point in the variable space with a simple function, which reasonably reflects the behavior of the objective function in a neighborhood region around the current point. This neighborhood is called the trust region. A trail step to find the optimal solution is to compute SSE according to Equation (10) and if the SSE is smaller than the existing minimum of SSE, the current point is updated; otherwise the current point remains unchanged and the trail step is repeated using a new neighbor point. More details about this algorithm can be found in the studies of More and Sorensen *(17)* and Coleman and Li *(18)*.

The Trust-Region algorithm can be easily implemented in many software applications. In our study, it was implemented using the optimization toolbox of Matlab 7.0. Compared to other parameter estimation methods, this approach is very easy to understand and straightforward to apply.

5. MODEL COMPARISON AND DISCUSSION

Using the sample dataset collected from the multiple-lane freeway section at the ES-167D station on southbound I-5, numerical experiments were conducted to examine the commonly used headway distribution models. Two popular measuring criterions to examine goodness-of-fit are the Chi-square test and Kolmogorov-Smirnov test. The Chi-square test, however, is not a very forgiving analysis and a model may be thrown off by only a few "bad" fits *(4)*. In this study, the Kolmogorov-Smirnov (*K-S)* test was chosen to measure goodness-of-fit of the selected headway models to the observed headway data. Additionally, a Quantile-Quantile (*Q-Q*) plot was provided to visualize the goodness-of-fit for each model. A *Q-Q* plot displays the quantiles of the sample data versus theoretical quantiles of a specific distribution. If the proposed distribution is consistent with the characteristics of the sample data, the plot will be close to a line.

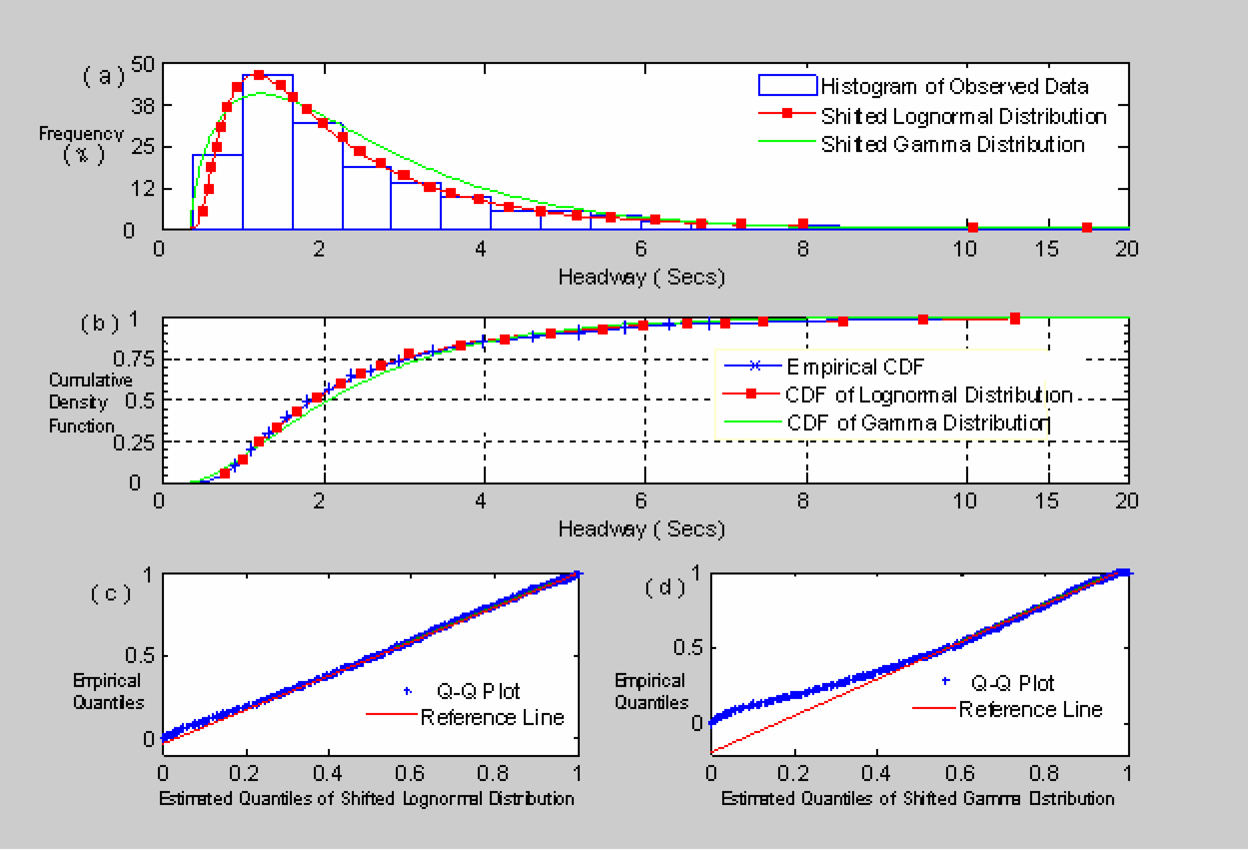

FIGURE 1 Single distribution model of 3-parameter shifted lognormal and shifted Gamma distribution fitted to headway data collected on Lane 2 in the evening period.

(a) Probability density function, (b) Cumulative density function, (c) Q-Q plot for shifted Lognormal distribution; (d) Q-Q plot for shifted Gamma distribution.

Firstly, two typical single distribution models, shifted lognormal distribution and shifted Gamma distribution, were investigated. Their parameters were estimated by the MLE technique. Due to the wild variations of traffic flow condition across time periods, headway models were specifically calibrated for each time period in which the flow conditions were relatively consistent. Also, performances of the selected headway models were examined for each time period. Figure 1 representatively compares the performance of those two single distribution models for the evening period for Lane 2. The curves describing the probability density function and *Q-Q* plot indicate that the shifted lognormal distribution provided better fit to the sample data compared to the shifted Gamma distribution. The analytical *K-S* test supports our conclusion drawn from visual comparisons of the curves. Table 2 tabulates the analytical results of *K-S* test for the three-parameter shifted lognormal distribution and shifted Gamma distribution with a displacement of 0.35 second. The maximum *K-S* test statistic (absolute difference between the theoretical and empirical cumulative distributions) of the shifted lognormal distribution is less than 0.032 for headways collected on Lane 2. For the HOV lane (Lane 4) due to the special characteristics of traffic flows, the model performance degraded to some extent and the maximum *K-S* test statistic reaches 0.093. Further analysis indicated that although the shifted lognormal distribution is acceptable for the regular lane in different time periods, a rather large inaccuracy deteriorates the model's performance for the HOV lane.

TABLE 2 Comparisons of Goodness-of-Fit of Single Distribution Models

Lane No.	Period	Average Flow Rate (vphpl)	3-Parameter Shifted Lognormal Distribution	3-Parameter Shifted Gamma Distribution
			K-S test	*K-S* test
Lane2	Morning	1680	0.016	0.062
	Noon	1485	0.028	0.052
	Evening	1445	0.032	0.077
	Night	323	0.020	0.084
Lane4	Morning	1408	0.093	0.116
	Noon	430	0.080	0.077
	Evening	462	0.073	0.088
	Night	24	0.070	0.167

Then, the performance of Cowan M3 was investigated against the field data. Figure 2 shows test results of this model for Lane 2 in the noon period at the flow level of 1485 vph. The best value for the constant headway τ is 1.21 second for our data. There is very little difference between the two curves generated from the two parameter estimation methods of ME and MLE. Both provided the almost identical overall goodness-of-fit. As can be seen in Figure 2, this model fits larger headways better than shorter headways possibly due to its simplified representation. The *Q-Q* plot reflects this situation clearly. For instance, the model shows no vehicle has a headway shorter than 1.21 seconds, while a significant number of vehicle headways are observed in this range. The *K-S* test statistics for Cowan M3 are relatively high as shown in Table 3, indicating less favorable fits over time periods at the test lanes. Although Cowan M3 has been used widely *(7, 8, 9, 10)* due to its simplicity, the model can not provide substantially good overall fits for the headway data collected for this study, especially for short headways. In addition to Cowan M3, Table 3 also shows the *K-S* test results for three other mixed distribution models: Cowan M4 with normally distributed following-vehicle headways, Cowan M4 with Gamma distributed following-vehicle headways, and DDNED. Examination results for these models are discussed as follows.

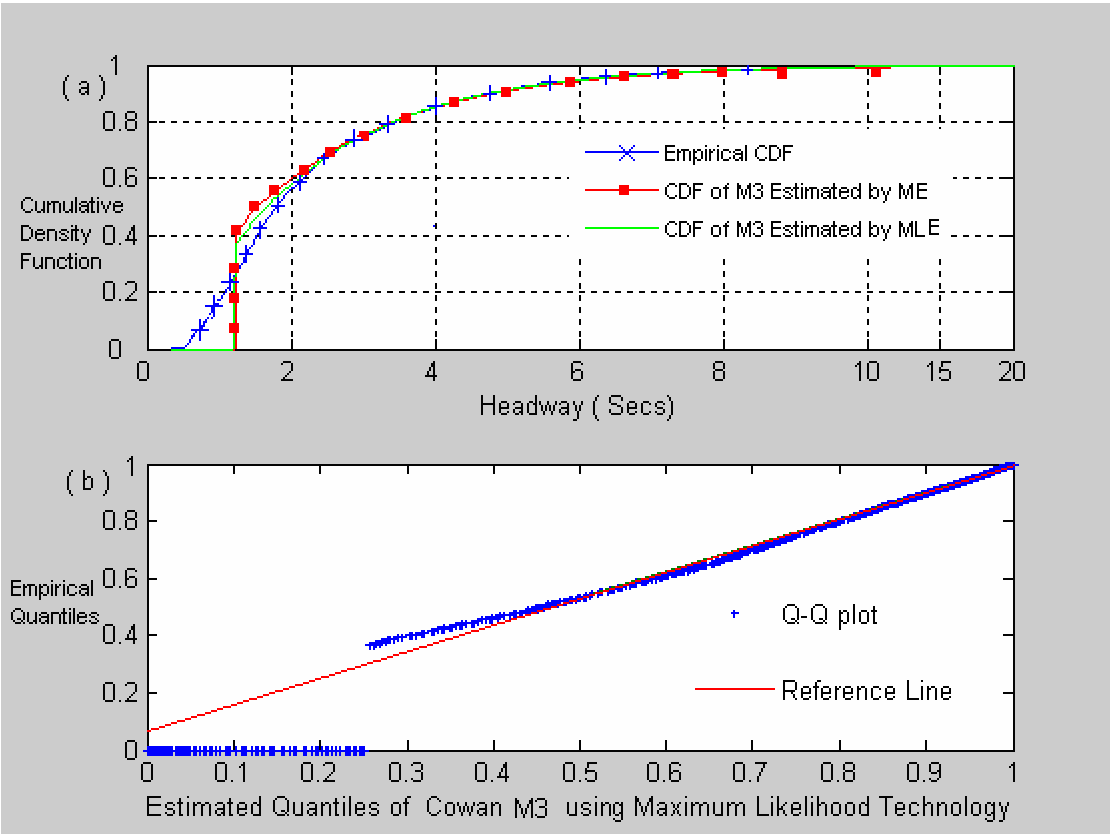

FIGURE 2 Cowan M3 fitted to headway data collected on Lane 2 in the noon period.

(a) Cumulative density function, (b) Q-Q plot for Cowan M3 estimated by ML technology.

Cowan M4 generalizes M3 to the extent that the following-vehicle headway is given a general distribution. In our study two underlying forms of following-vehicle headway distribution, the normal distribution and Gamma distribution were applied and examined with the model. The corresponding estimated parameters are shown in Table 4. For headway data collected on Lane 2 in the morning period, these two models performed well. The comparison curves of the cumulative density functions and *Q-Q* plot are shown in Figure 3. However, the overall performance of these two models is not favorable for other time periods on both Lane 2 and Lane 4. As shown in Table 3 the *K-S* test statistic for Cowan M4 with Gamma distributed following-vehicle headway is 0.265 for Lane 2, and even higher for Lane 4. Cowan M4 with normal and Gamma distributed following-vehicle headways did not demonstrate high goodness-of-fit to the headway data collected in this study.

TABLE 3 Comparisons of Goodness-of-Fit of Mixed Distribution Models

Lane No.	Period	Average Flow Rate (vphpl)	M3 Estimated Using ML	M4 with Following Vehicle Headways of Normal Distribution	M4 with Following Vehicle Headways of Gamma Distribution	DDNED
			K-S test	*K-S* test	*K-S* test	*K-S* test
Lane2	Morning	1680	0.178	0.125	0.087	0.084
	Noon	1485	0.208	0.208	0.152	0.030
	Evening	1445	0.187	0.265	0.147	0.032
	Night	323	0.217	0.361	0.265	0.021
Lane4	Morning	1408	0.292	0.328	0.169	0.079
	Noon	430	0.420	0.489	0.266	0.018
	Evening	462	0.380	0.767	0.233	0.017
	Night	24	0.611	0.767	0.770	0.069

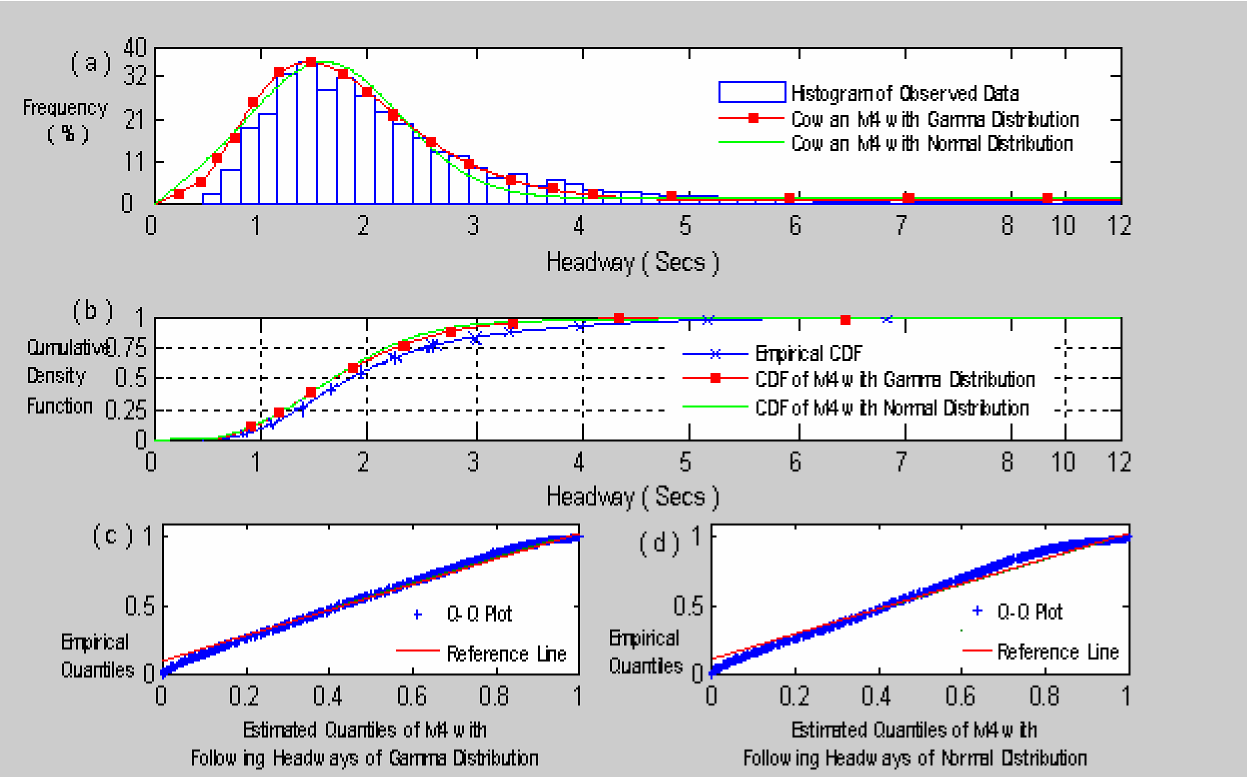

FIGURE 3 Cowan M4 with following headway of Gamma and normal distribution fitted to headway data collected on Lane 2 in the morning period.

(a) Probability density function, (b) Cumulative density function, (c) Q-Q plot for M4 with following headways of Gamma distribution; (d) Q-Q plot for M4 with following headways of normal distribution.

Another important mixed distribution model examined in this study is the DDNED model. Figure 4 illustrates the DDNED model's goodness-of-fit for Lane 2 at the noon and night periods which correspond to flow levels of 1485 vph and 323 vph, respectively. Figure 5 shows the same comparisons for Lane 4 at the flow levels of the 1408 vph (morning) and 462 vph (evening). Those sample data used in this examination covered various traffic flow levels from very low to reasonably high at the test location. The *K-S* test results shown in Table 3 indicate an encouraging performance of the DDNED headway model, especially, for the HOV lane. Due to the unique traffic flow pattern on the HOV lane, previous models examined earlier did not provide a close approximation to the headway distribution of the HOV lane. However, the DDNED model performed extremely well for fitting our sample data collected from both Lane 2 and Lane 4. In Table 3, the maximum *K-S* test statistic is less than 0.084 that is far lower than those of other models. The curves shown in Figure 4 and Figure 5 are consistent with the *K-S* test statistics.

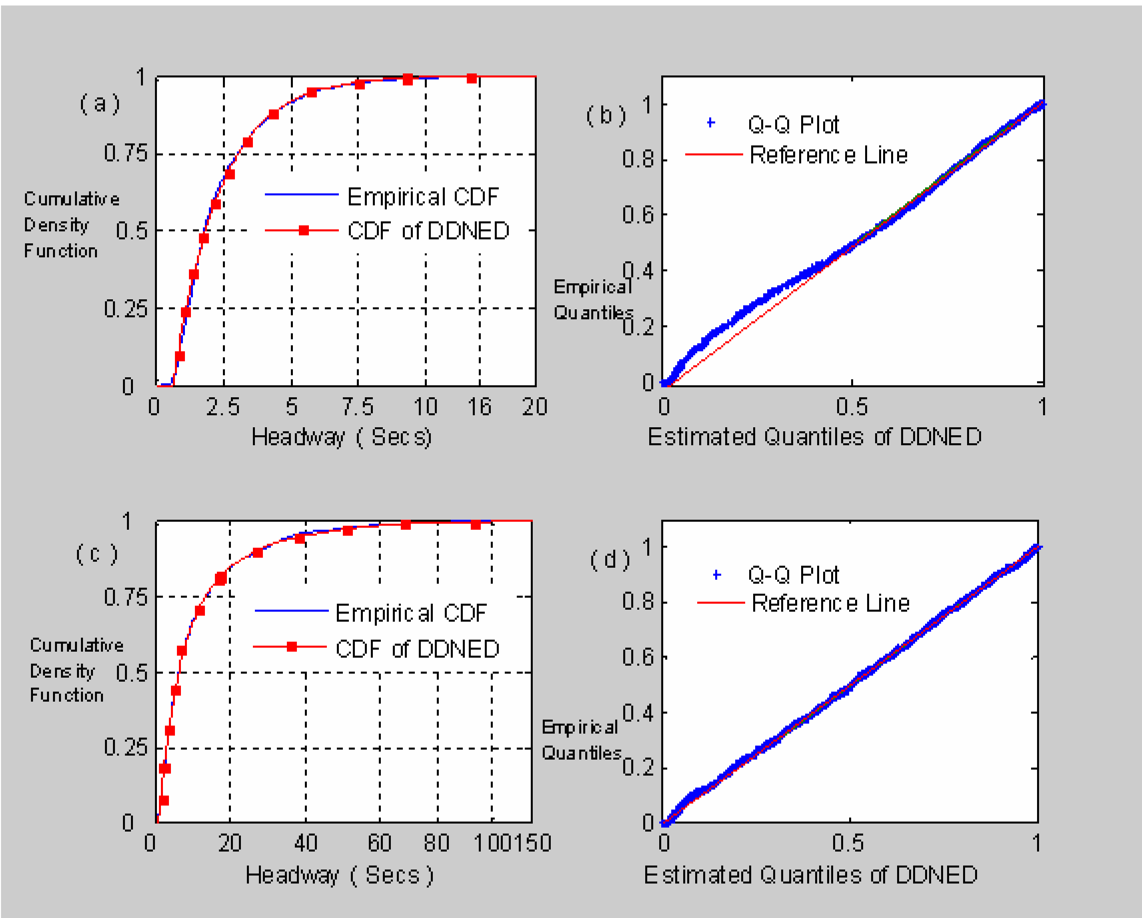

FIGURE 4 Cumulative density functions and Q-Q plot of the DDNED model fitted to headway data collected on Lane 2 in the noon and night periods.

(a) Comparisons of CDF of DDNED and empirical CDF for Lane 2 in the noon period,(b) Q-Q plot in the noon period, (c) Comparison of CDF of DDNED and empirical CDF for Lane 2 in the night period; (d) Q-Q plot in the night period.

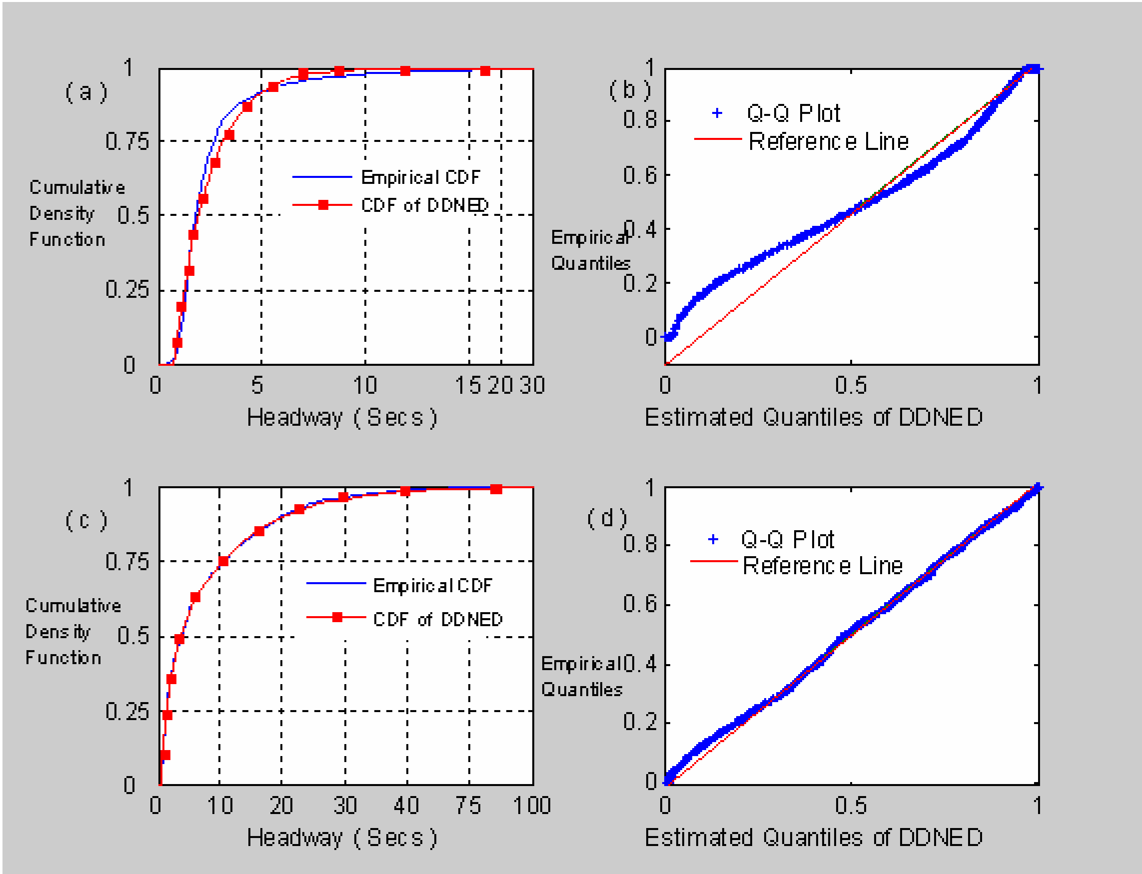

FIGURE 5 Cumulative density functions and Q-Q plot of the DDNED model fitted to headway data collected on Lane 4 in the morning and evening periods.

(a) Comparisons of CDF of DDNED and empirical CDF for Lane 4 in the morning period,(b) Q-Q plot in the morning period, (c) Comparison of CDF of DDNED and empirical CDF for Lane 4 in the evening period; (d) Q-Q plot in the evening period.

In general, our examinations on the typical headway distribution models show that both single and mixed distribution models can provide reasonably good fits to the headway data collected from regular lanes under different flow levels and time periods. For example, for the general purpose lane, in the noon, evening and night periods the shifted lognormal distribution and the DDNED models both fitted the headways very well, and yet in the morning period the shifted lognormal distribution outperformed the DDNED model slightly. However, due to the unique characteristics of headways on the HOV lane, most headway models performed less favorably in our examinations. As an exception, the DDNED model outperformed its peers. It performed extremely well for modeling both regular lane headways and HOV lane headways.

TABLE 4 Estimated Parameters for Each Mixed Distribution Model

Lane No.		Lane2				Lane4			
Period		Morning	Noon	Evening	Night	Morning	Noon	Evening	Night
Cowan M4 with Normal Distributed Following Vehicle Headway	θ	0. 159	0.250	0.261	0.002	0.347	0.416	0.020	0.370
	μ	1.585	1.289	1.314	0.424	1.419	0.135	0.434	0.734
	δ	0.719	0.782	0.848	0.004	0.638	0.851	0.025	1.101
	λ	0.004	0.007	0.007	0.231	0.007	0.100	0.166	0.645
Cowan M4 with Gamma Distributed Following Vehicle Headway	θ	0. 164	0.258	0.258	0.348	0.361	0.379	0.097	0.695
	α	4.998	3.204	3.260	2.413	4.993	0.934	4.619	1.439
	β	0.362	0.512	0.509	2.703	0.323	1.253	0.110	0.570
	λ	0.002	0.004	0.004	0.214	0.004	0.061	0.187	0.021
DDNED	θ	0.595	0.590	0.497	0.626	0.261	0.297	0.378	0.316
	λ_1	0.702	0.569	0.546	0.197	0.506	1.263	0.784	0.066
	λ_2	0.712	0.563	0.495	0.052	0.616	0.090	0.092	0.009
	d	0.775	0.621	0.624	0.799	0.785	0.582	0.611	0.263

6. CONCLUSIONS

Knowledge of headway distribution is very important for traffic flow theory and simulation research. Various single and mixed distribution models were proposed to adapt to varying traffic situations. Based on the vehicle headway data collected by the ALEDA system at an urban freeway section, the adaptability and accuracy of several typical headway distribution models were examined. Our examinations indicate that there is a significant difference in headway patterns between the regular lane and the HOV lane over different time periods of day. A total of six headway models, including two single distribution model and four mixed distribution models, have been investigated to test their goodness-of-fit to the sample data. In order to test the goodness-of-fit of the headway models, the *K-S* test statistics and visualized *Q-Q* plots were applied for visual and quantitative comparisons. While each model possesses some strength and weakness under certain conditions, in our examinations the shifted lognormal distribution was found to be adequate in fitting headways on general purpose lanes under most circumstances. But it was incapable of describing headway characteristics on HOV lanes. The DDNED model performed better than its peers for modeling both regular lane headways and HOV lane headways. Our test results provide helpful information in the selection of suitable headway distribution models for traffic flow theory research and simulation studies under certain traffic conditions.

Additionally, a new parameter estimation method was developed for complicated headway models. Compared to other parameter estimation methods, this method is easy to apply and estimation results are reasonable.

Our research findings on examined headway models are general. These findings and recommendations may be appropriate for interstate highways in an urban setting similar to that of Seattle. Before the models are exploited in other areas, however, we suggest using a small amount of headway data to re-examine their spatial transferability.

There are two directions for further research work. First, because of the unique characteristics of traffic flow on the HOV lane, more data need to be collected and analyzed to identify the associated traffic flow patterns with the HOV lane. Second, the Cowan M4 model is generally considered more realistic and flexible. Its unfavorable performance in our examinations may be due to the inappropriate headway distributions of the following vehicles. We recommend that more following-vehicle headway distributions should be tried for a thorough investigation to Cowan M4.

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