MODELING TRAFFIC FLOW DYNAMICS ON MANAGED LANE FACILITY: A CELL TRANSMISSION MODEL BASED APPROACH

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ABSTRACT

With the increasing attention paid over environmental impacts and sustainable infrastructure, transportation agencies are now seeking for more cost-effective and environmental friendly countermeasures against traffic congestion than the traditional roadway expansion. Managed lane (ML) systems, as an innovative strategy for managing the roadway conditions in real time, have been gaining increasing popularities in the recent decade. However, the unique characteristics of ML facilities are not well studied and modeled, such as frictional effects between the General Purpose Lanes (GPLs) and their adjacent ML. This paper investigates the interaction between GPLs and MLs, as buffer-separated ML facilities are readily impacted by congestion in the adjacent GPLs. The frictional effect is quantified through the development of speed-flow curves for the ML facility. A traffic flow model is developed on the basis of Cell Transmission Model (CTM) incorporating this frictional effect to model the traffic evolution on the ML facility. This model is capable of replicating the traffic flow pattern, including congestion onset, propagation and dissipation at a macroscopic level. It also captures the impact of GPLs congestion onto the ML. This model can be served as underlying ML traffic flow model for evaluating the effectiveness of different tolling strategies in the future research.

KEYWORDS: Cell Transmission Model, Managed Lane, Friction Effect, Macroscopic Simulation, and Traffic Flow Theory

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INTRODUCTION

Traffic congestion is getting worse nationwide, especially in the urban area. Transportation agencies are therefore seeking for alternatives to mitigate the negative impacts of traffic congestion other than roadway expansion. One of the solutions towards this problem is the concept of Managed Lane (ML). MLs provide a good opportunity for improving person-throughput within the existing lanes, by reserving one or more lanes to special categories of users. The restrictions imposed on user groups often include the number of occupants, types of vehicles, and tolling *(1)*. While there are more and more ML facilities in operation nationwide, the operational experience for ML is minimal, especially in terms of the methodology or modeling technique to quantify the performance of MLs as well as appropriately modeling its interaction with the adjacent General Purpose Lanes (GPLs).

In comparison with analytical solutions, traffic simulation platform offers more flexible and user friendly approach to ML system performance analysis. More importantly, simulation tools can be used to assess the performance of real-time traffic control and traffic management under the intelligent transportation system (ITS) context. Simulation models have been evolved to fit different research needs and project scopes and are distinguished by the desired level of details for the representation of the real-world scenario. Simulation models are often classified into three different categories: microscopic, mesoscopic and macroscopic based on the level of details one requires. Microscopic traffic simulation models assume that the behavior of an individual vehicle is a function of the traffic condition in its environment *(2)*. While macroscopic simulation assumes that the aggregate behavior of a set of vehicles, which is easier to observe and validate, depends on the traffic conditions in their environment. It utilizes the fundamental relationship among speed, flow and density, and also requires less computational resources than microscopic simulation. Mescoscopic model is a mixture of both, which often times incorporates the movement of platoons of vehicles and their interaction *(3)*.

The Cell Transmission Model (CTM) is a discrete version of hydrodynamic theory of traffic flow and has been widely recognized to simulate traffic flow evolvement under various conditions due to its analytical simplicity and ability to reproduce congestion wave propagation dynamics. It was developed by Carlos Daganzo in 1993 as a solution to the differential equations of the Lighthill, Whiteman, and Richards (LWR) hydrodynamic model for the representation of traffic flow *(4, and 5)*. The basic CTM assumes a piecewise two-wave flow-density relationship. Because of its macroscopic features, it offers calibration and computation advantages over microscopic simulation models. The original CTM developed by Daganzo *(2)* was used to examine the evolution of traffic over a one-way road without any intermediate access points. So vehicles enter at one end and leave at the other.

In this paper, the interaction between the ML and GPLs is explored. This interaction is referred to as Frictional Effect in the paper, which defines as the degradation in performance of a ML facility due to a poor performance on the adjacent GPL facility *(6)*. A CTM-based traffic flow model is developed to integrate this interaction for modeling ML facility for simulation purposes. The results are of crucial value for functioning as an underlying traffic flow model of ML facility in evaluation of the performance for freeway segment with MLs.

RESEARCH OBJECTIVES

The major goal of this research is to better model and elaborate the traffic evolution on ML facilities by incorporating a frictional adjustment factor into the original CTM. It is proved from

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previous research that the interaction between the two lane groups (GPL vs. ML) within the freeway segment would impose a friction effect on the ML, so that even if the ML does not reach its capacity, a speed reduction would still be observed. By modeling the two lane groups simultaneously within a freeway segment, this model would be capable of accurately replicating the actual traffic evolution on both ML and GPLs, as well as retaining the interaction between them. Specific objectives of the research include:

1. Quantifying the frictional intensity factor from field collected data into the speed-flow relationship model;
2. Building a traffic flow model on the basis of CTM incorporating the frictional effect imposed onto the ML facility due to the GPLs' slow-down traffic on the MATLAB platform; and
3. Evaluating the performance of the developed simulation model by performing traffic flow pattern analysis.

To precisely reflect ML facility traffic operational attributes, field data were collected to determine the frictional effect as well as the flow-density relationship. Based on ground-truth data collected under NCHRP 03-96 research project, Analysis of Managed Lanes on Freeway Facilities, this study is able to develop an approach on the basis of the original CTM to simulate the traffic evolution on ML facilities at a macroscopic level.

PREVIOUS STUDY

The design of ML facilities is considered an important issue associated with the performance of ML throughput, travel time and safety *(1)*. Some of the key design factors include separation type, number of lanes, and access points. Due to different configurations, the traffic flow characteristics of ML could be quite different from that of GPLs. High-Occupancy Vehicle (HOV) lane is one classical type of ML facility. It is commonly adopted nationwide to improve person-throughput on the freeway. Most HOV lanes are left-concurrent, running parallel to the GPLs *(1)*. They are often separated from the GPLs by single white line, so that vehicles can ingress and egress at any location. It is often observed that HOV lanes do not operate at expected speeds relative to volume under this design *(7)*. It was found that when GPLs move slowly, it tends to have a frictional effect to slow down HOV lane as well. The reasons for this frictional related slowing were identified as motorists lack of comfort traveling at a large speed differential and that vehicles merging from the ML to the GPL had to slow down *(8)*. Guin *et al. (9)* also investigated the traffic behavior associated with the interaction of parallel, buffer-separated HOV and GPLs along I-85 in Atlanta, GA. Their results indicated that the speed differential between the HOV lane and the adjacent GPL is affected by the level of congestion in each lane. When the GPL is uncongested, there is no speed differential between the two systems. When the GPL is congested, the HOV lane's speed is higher than the GPL's speed. This maximum speed differential between the facilities was between 20-30 mph. The speed differential gradually decreased until both facilities reached a state of equal congestion. Cassidy *et al. (10)* further proves that an HOV lane on congested freeways demonstrates a smoothing effect to the overall roadway by significantly increasing the bottleneck discharge flow in the adjacent GPLs. It is mainly because of the fact that disruptive lane changes diminish due to the presence of HOV lane. Liu *et al. (6)* later investigated the frictional effect between GPLs and MLs across different separation types. The frictional effect of buffer-separated facilities appeared to be the greatest.

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They were followed by soft barrier (plastic pylon) and concrete barrier separation. Concrete barrier separation appeared to have no statistically significant frictional effect.

It is noted that the ML facility operates differently than the GPL facility due to the interaction between the two lane groups (ML vs. GPL). Therefore, when modeling a ML facility, it would be imperative to take into account and quantify this interaction so that the freeway can be appropriately represented. Following Liu et al. *(6)* study, the interaction between GPL and ML is referred to as "frictional effect", in that due to the proximity of GPL and ML traffic, increasing congestion levels on GPLs are proved to have an adverse effect on ML operations, well before the ML demand reaches breakdown levels. This effect is particularly significant at single lane buffer-separated ML facilities, where a single ML running parallel to the GPLs and is separated from the GPLs with a 2-4 foot wide striped buffer. Vehicles are only allowed to access the ML at designated access point. The reason for this frictional effect is that drivers in the MLs can readily observe the traffic on the adjacent lanes, and feel uncomfortable passing congested GP traffic at a high speed differential without adequate barrier separation.

The CTM is adopted in this study for modeling the two lane groups in the ML facility due to its analytical simplicity and ability to reproduce congestion wave propagation dynamics. The CTM has been previously validated for a single freeway link using data from I-880 in California *(11)*. The model can be described by just using three of the four fundamental freeway parameters such as capacity, jam density, free-flow speed and backward wave speed. In CTM, freeway is divided into homogeneous cells and the length of each cell is determined such that all vehicles in one cell will flow into the downstream cell in one time step under free-flow conditions *(12)*. However, when queue is formed, the simulation will be based on a recursion where the number of vehicles (cell occupancy) in each cell at time *t+1* equals its occupancy at time *t*, plus the inflow and minus the outflow. The following expression therefore stands:

$$n_i(t+1) = n_i(t) + y_i(t) - y_{i+1}(t) \tag{1}$$

where $n_i(t+1)$ is the number of vehicles in cell *i* at time *t+1*. $y_i(t)$ is the inflow of cell *i* at time t. $y_{i+1}(t)$ is the outflow of cell *i*, which is also the inflow of downstream cell *i+1*.

In each cell, CTM assumes a piecewise linear relationship between flow and density. It is depicted in Figure 1 and expressed as:

$$q = \min\{vk, Q, w(K-k)\} \qquad 0 \le k \le K \tag{2}$$

where v=free-flow speed of the freeway segment
k=density of the freeway segment
K=the jam density
Q=the maximum flow rate (capacity), and
w= the backward wave moving speed.

If Equation (2) is replaced into the flow conservation equation, differential equation that would define the evolution of traffic under the LWR model can be obtained:

$$\frac{\partial \min\{vk, Q, w(K-k)\}}{\partial x} = -\frac{\partial k}{\partial t} \tag{3}$$

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The flow advancing equation can be correspondingly derived as *(2)*:

$$y_i(t) = \min\{n_{i-1}(t), Q_i(t), \delta \cdot [N_i(t) - n_i(t)]\} \tag{4}$$

where $n_{i-1}(t)$ is the number of vehicles in cell i-1 at time t; $Q_i(t)$ is the maximum number of vehicles that can flow into cell *i* in one time step (capacity); $N_i(t)$ is the maximum occupancy of cell i. $\delta = w/v$.

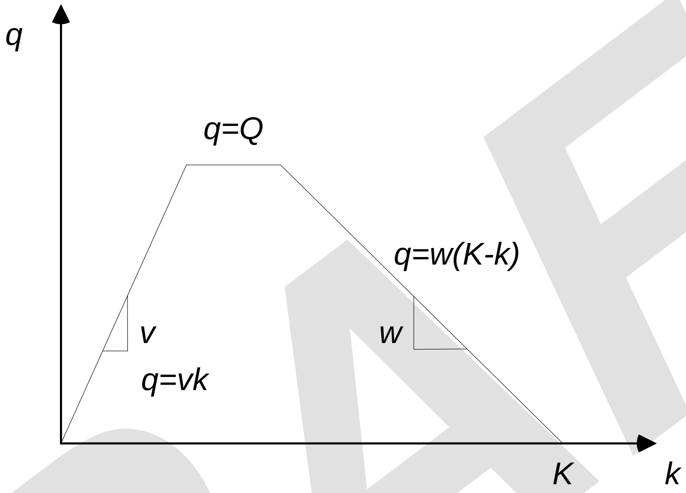

FIGURE 1 Flow-density relationship for the basic CTM.

METHODOLOGY

In this study, the CTM is primarily modified to suit for the buffer-separated single-lane ML facility. Previous efforts *(7)* have identified that traffic operation on this type of facility differs from that on GPLs. The differences identified can be summarized into three categories:

1) Lower Capacity –The capacity of ML is lower than it would be expected on GPLs under the same demand.
2) Affected by the inability to pass slow moving vehicles – The inability for slow moving vehicles to be passed in the ML can result in slower facility speeds.
3) Affected by congestion in GPLs – Vehicles on the ML will slow down to reduce the discomfort of traveling at high difference in speed.

Among these three categories, items (1) and (2) are quite pronounced for this type of ML design. Item (3), however, is often referred to as “frictional effect” and was observed from several studies *(1, 7, and 10)*. The frictional effect can first be observed by viewing speed-flow plots for a buffer-separated ML facility in Figure 2. In Figure 2a, all the speed-flow data points are shown. There is a wide dispersion of points from the flow of 600 passenger car per hour per lane (pcphpl) to 1200 pcphpl. This trend is atypical for what is usually observed in GPL speed

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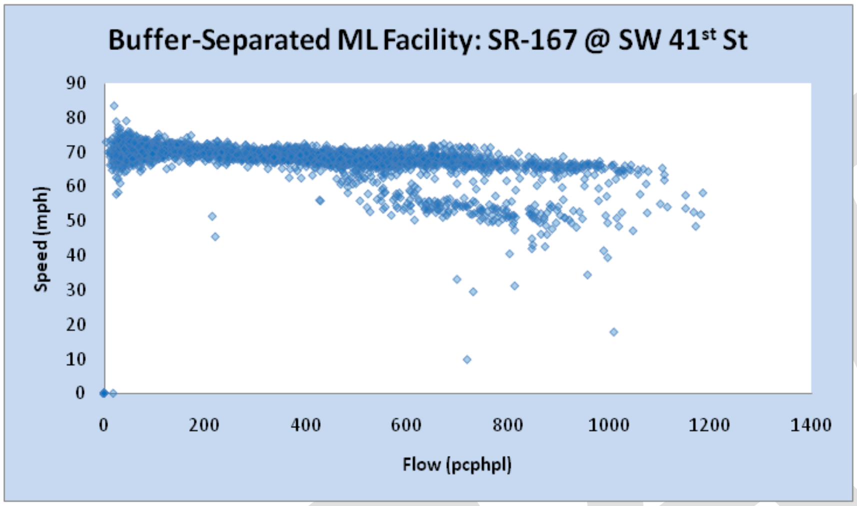
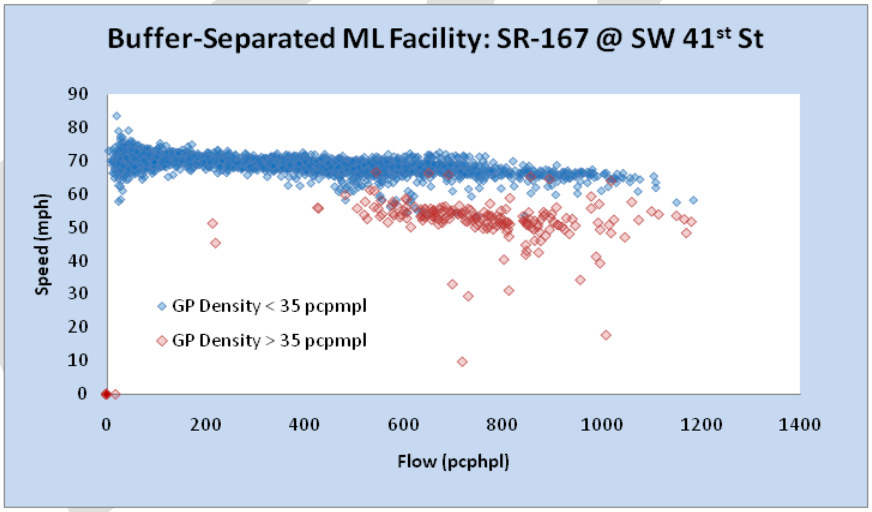
flow curves, which normally has little variance in speed between equal flows. When the data is segregated based on the GPL's condition, the reason for the dispersion of speed can be readily seen. Here, a density of 35 passenger car per mile per lane (pcpmipl) is used as a threshold for the drop in performance of the GP facility. This particular threshold was selected as it serves as the transition point from LOS D to LOS E for performance measurement of freeway facilities in the Highway Capacity Manual (HCM) 2010 *(16)*. During periods of time where the GPL has a density greater than 35 pcpmipl, lower speeds can be observed on the ML. The lack of continuity to the curve can be seen as breakdown appears to occur at different levels of flow depending on the GPL's performance.



(a) All ML Data Points



(b) Paired ML Data Points by GP Density

FIGURE 2 Speed-flow curve of buffer-separated ML facility.

On the basis of NCHRP 03-96 research project, the formulation of the set of speed-flow curves for the buffer-separated single ML facility is developed and shown in Figure 3. For details, please refer to the study by Thomson *et al. (7)*. It should be noted that for each FFS (free flow

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speed), there are two sets of curves to represent friction and non-friction scenarios, separately. That is because the performance of the buffer-separated facility is dependent on not only the characteristics of the ML but also the performance of the adjacent GPLs. The frictional curve, therefore, has been produced showing the speed-flow relationship for periods of time during GP congestion. The friction curves terminate at a density of 45 pcpmipl, consistent with the methodology used in HCM 2010. The range of observed data for the non-friction curves never reached such high density levels, probably attributable to a low likelihood of observing non-friction cases in combination with high flow rates. As a result, the terminal density of the non-friction curves is 30 pcpmipl.

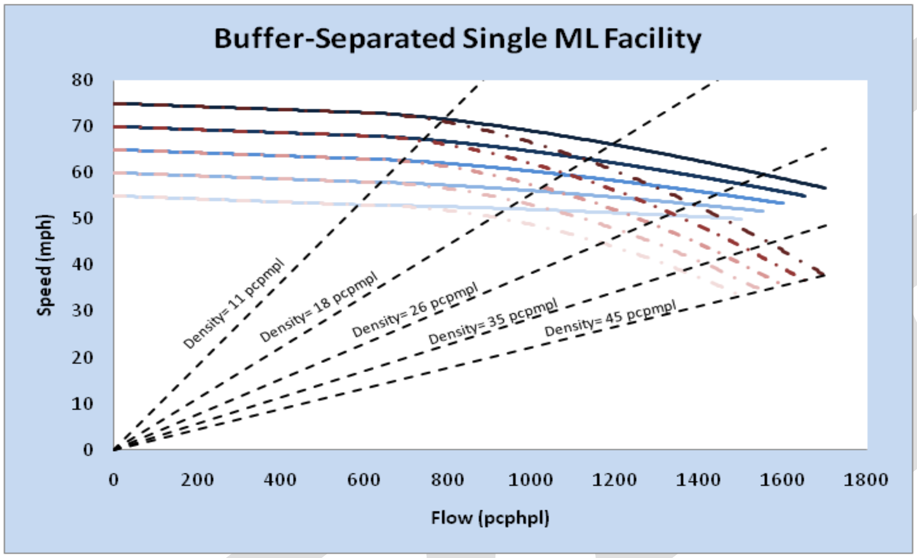

FIGURE 3 The speed-flow curve family for buffer-separated single ML facility.

The developed speed-flow curves for ML facility combining with the GP speed-flow relationship published in HCM 2010 function as the basis for the proposed traffic flow model utilizing CTM. The CTM is applied here to model the one-way traffic on the ML freeway segment. A cell representation of the freeway segment of interest is shown in Figure 4. The freeway segment, exploited as a single unidirectional link in this study, is homogeneous. This segment is simulating the SR 167 HOT lane system in Seattle, Washington, where a single ML is separated from two GPLs by a double white line buffer. There are several access points along the SR 167 HOT lane corridor. However, in this study, we only focus on the basic segment where travelers are not allowed to switch lanes between ML and GPLs. The FFS on this segment is 65 mph from the empirical data.

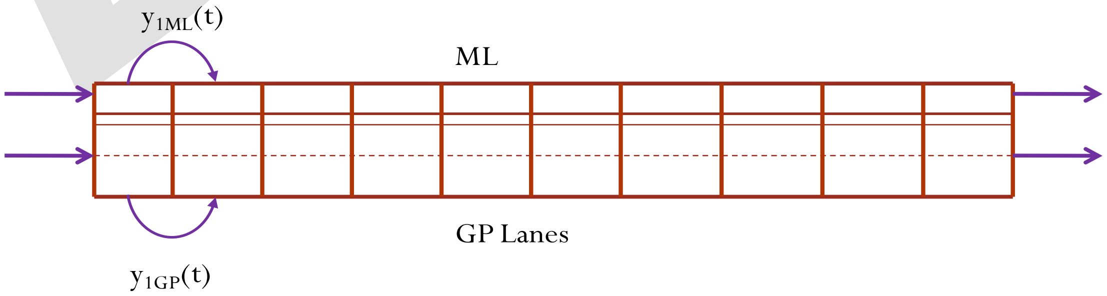

FIGURE 4 Cell representation of buffer-separated ML facility.

Under the HCM context, the parameters for the flow-density relationship characterizing the link can be calibrated. The determination of the capacity and jam density for the GPLs and buffer-separated ML is based on the calibrated curves provided in HCM 2010 and NCHRP 03-96 research project. The lower portion of the curves is regressed using Van Aerde's steady-state car-following model *(13)*. Figure 5 shows the flow-density curves for both the ML and GPLs. At the SR 167 HOT lane corridor, the FFS is determined to be 65 mph for the freeway link. On the GPLs, the capacity is chosen to be 2300 vphpl, and the jam density is approximately 165 vpmipl. Based on the assumed triangular shape of flow-density relationship, the backward wave speed is 17 mph. For the ML, when no frictional effect is present, the capacity can reach up to 2000 vphpl, and the jam density is about 160 vpmipl. The backward wave speed is therefore about 15 mph. However, when frictional effect exists, the capacity reduces to 1600 vphpl, and the jam density is only around 125 vpmipl. The corresponding backward wave speed is 16 mph.

The total length of the freeway segment analyzed in this study is 6.5 mile. The free-flow travel time is then 0.1 hour. The parameters of the CTM are set as shown in Table 1. The traffic advancing logic on the GPLs will follow Equation 4. While for ML, the flow advancing to next cell will also be determined by its neighboring GP cell condition. If the neighboring GP cell is under congested condition (>35 vpmipl), then the inflow to the next ML cell will use the set of parameters for the "friction" state. Accordingly, if the neighboring GP cell is in the uncongested scenario, then the flow advancing to the next cell will adopt the parameters of the "non-friction state".

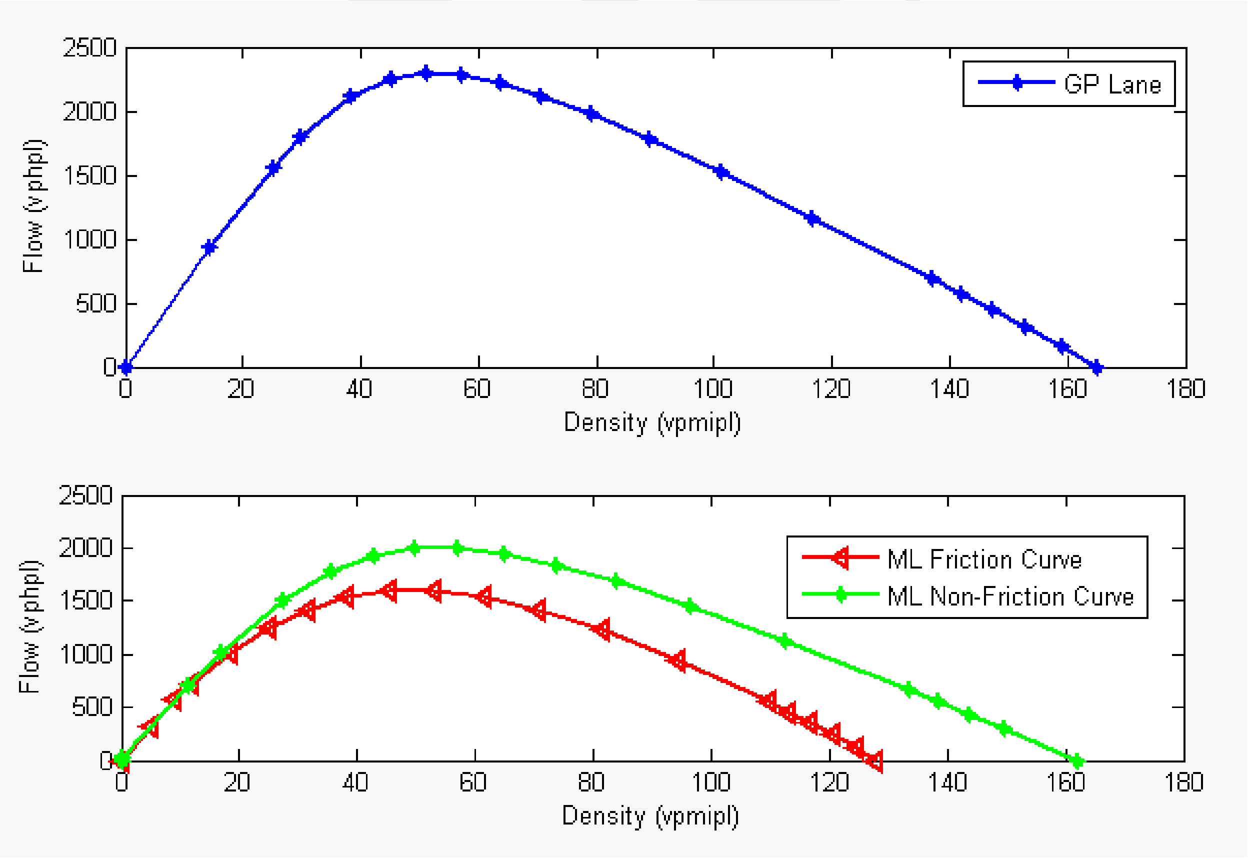

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FIGURE 5 Flow-density curves calibrated for GPLs and buffer-separated ML facility.

TABLE 1 CTM Parameters for the Freeway Segment

Time Step	**0.01 hour**		
Cell Length	**0.65 mile**		
	GPLs	**ML**	
		Non-Friction	**Friction**
Jam Density	**165 vpmpl**	**160 vpmpl**	**125 vpmpl**
Backward Wave Speed	**17 mph**	**15 mph**	**16 mph**
δ	**0.26**	**0.23**	**0.25**
Maximum Inflow for Each Cell	**23 veh**	**20 veh**	**16 veh**
Maximum Occupancy for Each Cell	**107 veh/lane**	**104 veh**	**81 veh**

Figure 6 demonstrates the framework of the proposed traffic flow model incorporating both GPL and ML lane groups. Note that upon the update of the inflow for each cell at each time step, the occupancy of the current cell (number of vehicles in each cell) would be updated as well following Equation 1.

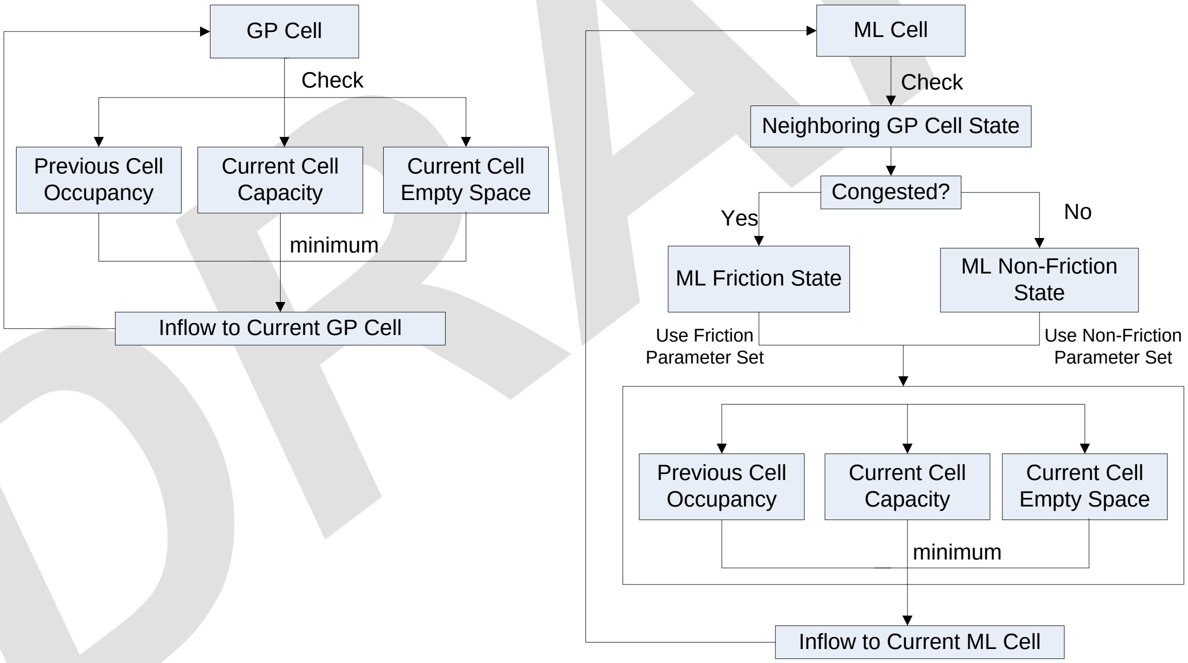

FIGURE 6 Framework of the traffic flow model.

RESULTS

In the proposed traffic flow model on the basis of CTM, boundary conditions are specified by means of input and output cells according to Daganzo *(2)*. The output cell, functioning as a sink to absorb all the exiting traffic, has infinite size (set as 10000000 in this study). A source cell also with an infinite number of vehicles is set that would discharge into a “gate” cell of infinite

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size. This source cell functions as a valve that would release the inflow traffic at the default rate while holding the rest traffic flows that cannot enter the cell.

The relevant parameters in the model were either taken from the empirical study or estimated based on the actual condition of SR 167 freeway sections *(15).*

With the developed model, a numerical study is performed to observe the inflow patterns within the simulated freeway segment. For a simulation period of 100 time steps (1 hour period), the GP volume that enters into the segment is changed from 2000 to 2800 veh/hr/lane with a 200 veh/hr/lane increment for every 20 time steps (0.2 hour). The ML volume that is trying to enter the segment is set as 1800 veh/hr/lane constant for the entire period. Figure 7 shows the contour plot of traffic evolution (cell inflow) of the freeway segment for ML and GPLs, separately in both spatial and temporal dimensions. It is noted that the traffic will fill up the entire segment by the end of 10 time steps (0.1 hour) which is consistent with the assumption of the FFS = 65 mph. With the increase of the traffic flow trying to enter the GPLs, the GPLs are getting congested. After the 40th time step where the GP inflow (2400 veh/hour/lane) exceeds its capacity (2300 veh/hour/lane), the GP cells are unable to provide enough space for accommodating all the incoming vehicles. Therefore, for the rest of the simulation period, the largest flow that each cell could take is constrained to 23 veh/time step. For the ML cell, since it is affected by the status of the GPL, it is noted that although the inflow into the ML is constant, the actual flow passed on to each cell indeed varies. Beginning at the 40th time step, the adjacent GP cell was detected to have a density higher than 35 vehpmipl, and it thus imposed a frictional effect onto the ML. Correspondingly, the ML cell was shifted to the "friction state", where its cell capacity and maximum occupancy were decreased. The maximum inflow that it could take in for each cell is constrained to 16 veh/time step.

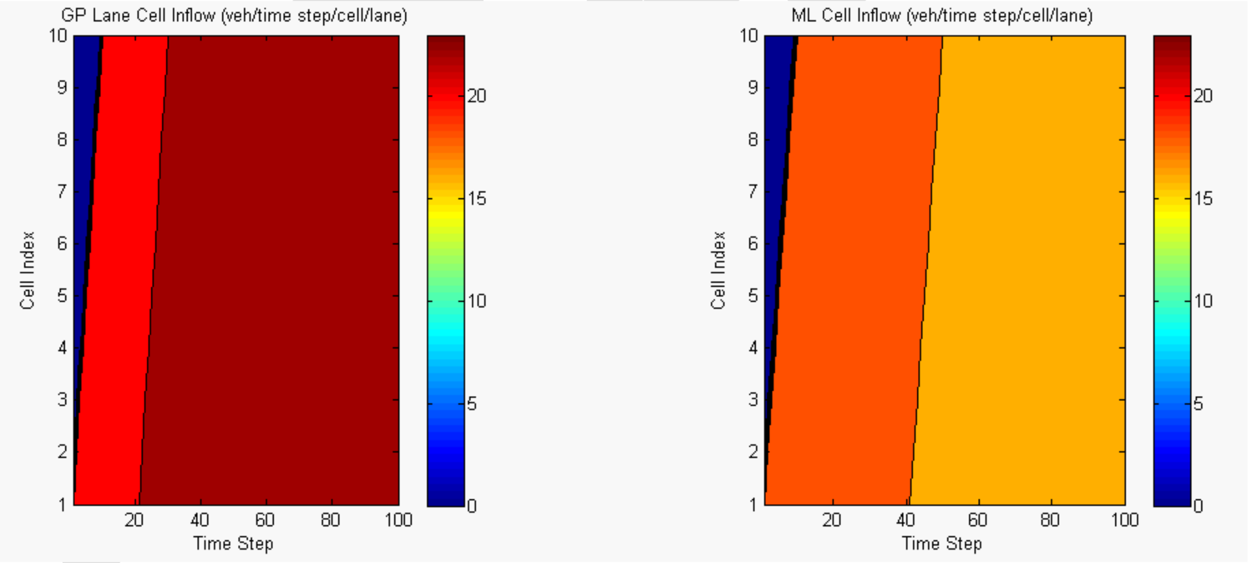

FIGURE 7 Temporal-spatial traffic evolution for the numerical study using the developed model.

To further model the freeway bottleneck scenario and better observe the process of onset, propagation, and dissipation of traffic congestion, a bottleneck GP cell (Cell 7) is set where the cell capacity (maximum inflow for the cell) is dropped by 60% during Time Step 20 through 30 to simulate an incident scenario. The source inflows for GPL and ML are set as 2000

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veh/hour/lane and 1800 veh/hour/lane, separately for the entire simulation period. Figure 8 shows the contour plot of traffic evolution pattern under the bottleneck scenario. The GP inflow pattern appears to be the well-known inverted triangular shape *(14)*. Essentially, upon the formation of a bottleneck at Cell 7, traffic gets slower downstream. This effect also propagates upstream, only to eventually disappear. After the bottleneck is cleared, the downstream cells are capable of absorbing the inflow traffic as much as they can that has been previously accumulated upstream. The ML cells performance is also impacted due to the bottleneck.

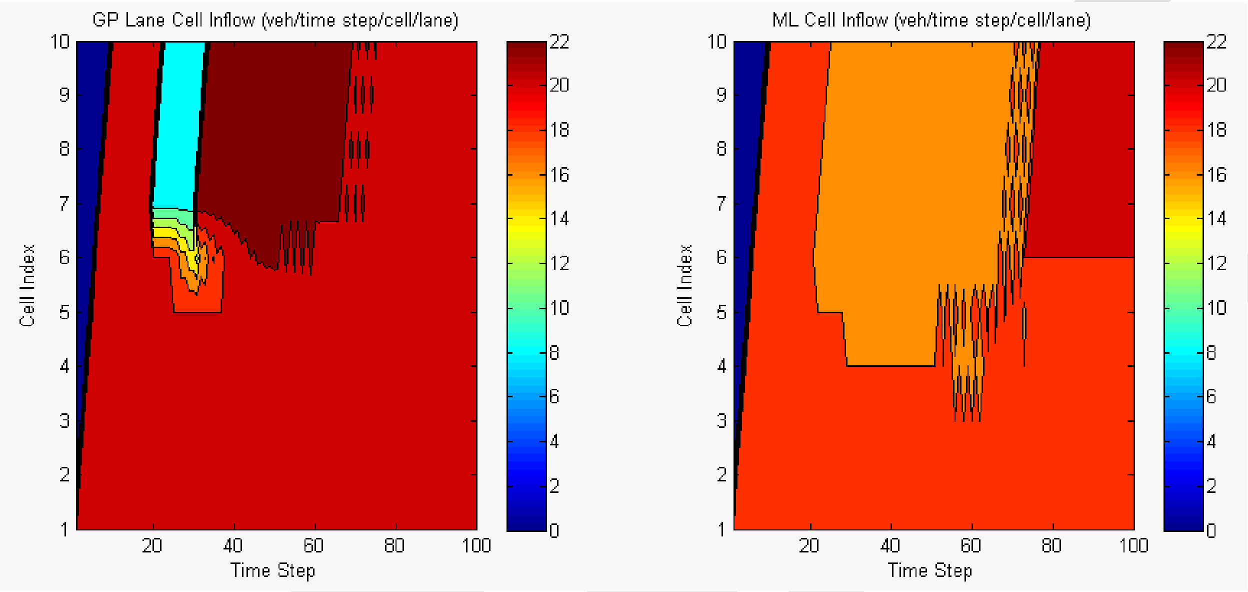

FIGURE 8 Temporal-spatial traffic evolution for the numerical study using the developed model under bottleneck scenario.

It is therefore demonstrated that the developed traffic flow model on top of the original CTM is capable of capturing the traffic evolution pattern on the current freeway segment as well as modeling the interaction between the GPLs and its adjacent ML. This model can function as the basis for evaluating the effectiveness of different traffic control strategies, such as congestion pricing algorithm on the ML facilities, etc.

DISCUSSION AND CONCLUSION

The concept of MLs has been widely adopted nationwide as a solution against traffic congestion. More and more ML facilities have been implemented in the freeway corridors. A lot of research has been done to study the ML performance from an operational perspective. However, few efforts have focused on investigating the traffic flow difference between GPLs and MLs as well as modeling the ML facility effectively.

Previous studies *(1)* proved that the congested GPLs would have an adverse effect on the adjacent buffer-separated ML, even when the ML itself is operating below capacity. This frictional effect is very important for developing methods to better estimate the operational performance of ML facilities. This paper, as a subsequence of this finding, developed a macroscopic simulation approach to quantify and incorporate this frictional effect for better modeling the traffic dynamics on the ML facility. Under the NCHRP 03-96 research project and HCM context, new speed-flow relationships for the ML facilities on the basis of different geometric conditions are developed. The speed-flow relationship for buffer-separated ML facility consists of two separate curves accounting for the influence of GPLs' congestion. Consequently, this research developed a traffic flow model on the basis of CTM to model the GPLs and ML lane groups separately while retaining the interaction between the two. The proposed model incorporating the frictional effect from the GPLs is capable of modeling the buffer-separated ML facilities that reflects the actual traffic evolution dynamics on the freeway segment.

This CTM-based modeling approach is important in the consideration for developing analytical procedure for ML evaluation in the HCM context. Due to its macroscopic nature, it is also less time-consuming and computationally efficient to implement for the practitioners than the microscopic simulation models. Future research will be performed to incorporate the modeling of ML facility access points, where weaving movement is involved. The developed model can also serve as the basis for evaluating the performance of buffer-separated ML facilities with various traffic inflows, and different tolling strategies. For example, the effectiveness of various congestion pricing strategies, such as time-of-day tolling and dynamic tolling on the basis of real-time traffic conditions of the two lane groups can be evaluated. Of course, certain assumptions need to be made in accordance with the actual scenario. Using dynamic tolling as an example, the utility functions that the travelers' lane choice model is based on need to be determined. The Origin-Destination (OD) data is preset, however, with the toll price being adjusted dynamically, the number of vehicles in each lane group would be varied from time to time. By evaluating the network-wide traffic conditions, the effectiveness of each tolling strategy can be determined.

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